HEAT TRANSMISSION BY NATURAL CONVECTION IN CYLINDRICAL AND SPHERICAL INTERLAYERS

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On the basis of the similarity theory, the law of heat transmission is derived in a form which allows extrapolation to the extreme values of parameters and, particularly, makes it applicable to a plane interlayer as well as to a cylinder (sphere) in infinite space.

A dimensional analysis of the system of differential equations and boundary conditions describing natural convection of gases and nonmetallic liquids ($Pr \gg 1$) in an enclosed volume yields, as is well known, one dimensionless group the Rayleigh number $Ra = (g\beta\Delta T/\nu a)h^3$, which uniquely defines the heat transmission by natural convection. Its cube root $Ra^{1/3}$ can be represented as a ratio of two linear dimensions: the interlayer dimension h and some scale dimension $(\nu a/g\beta\Delta T)^{1/3}$. In order to explain the physical meaning of the second dimension, we use the referred thickness of the thermal boundary layer (δ) and, on the basis of this concept, we write the orders of magnitude for the terms in the equation of heat balance and in the equation of motion:

$$a\frac{\Delta T}{\delta^2} \approx w\frac{\Delta T}{\delta}; \quad \nu\frac{w}{\delta^2} \approx g\beta\Delta T.$$

From these equalities we find that $\delta \sim (\nu a/g\beta\Delta T)^{1/3}$ or $h/\delta = CRa^{1/3}$, where C is a numerical coefficient and Ra is defined with respect to the thickness of the fluid interlayer h. In this way, the Rayleigh number represents the ratio of the total interlayer thickness to the thickness of the zone where heat transmission is effected by conduction. Inasmuch as instability and motion occur beginning at some critical value Ra_{cr} of the Rayleigh number, $C(Ra_{cr})^{1/3} = 1$. Therefore, $h/\delta = (Ra/Ra_{cr})^{1/3}$. This analysis makes it possible to interpret the law of heat transmission and its mathematical structure for interlayers of various geometries.

For a plane interlayer, the ratio $\alpha = \lambda/\delta$ renders this law in the form:

$$Nu = \left(\frac{Ra}{Ra_{cr}}\right)^{1/3},$$

where both Nu and Ra are defined in terms of the interlayer thickness h. For $Ra_{cr} = 2000\text{-}4000$, indeed, this law conforms to the many test data on natural convection in horizontal and vertical plane interlayers already when $Ra > 10{,}000$ [1, 3].

In writing the general relation $Nu = f(Ra)$ for a plane interlayer, we assume that for an interlayer of any geometry the ratio

$$\frac{h}{\delta} = f(Ra) \tag{1}$$

remains invariant.

Let us consider the transmission of heat through an interlayer between two cylinders with diameters d and D, respectively, whose thickness is $h = (D-d)/2$. Because of the asymmetry in convective flow, we need the dimensions of the zones where the heat transmission is effected by conduction: δ_1 and δ_2 at the inner and the outer cylinder, respectively, their sum equal to δ in Eq. (1). The ratio δ_1/δ_2, which depends

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on the hydrodynamic flow mode, will be assigned some numerical coefficient. Using the rules of adding thermal resistances, we find the following expression for the heat transmission coefficient:

$$\frac{1}{\alpha_{tot}} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2}, \tag{2}$$

where α_{tot}, $\alpha_1 = 2\lambda/d \ln[(d + 2\delta_1)/d]$, and $\alpha_2 = 2\lambda/d \ln[D/(D-2\delta_2)]$ refer to the surface of the inner cylinder. And, finally, we have the law of heat transmission by natural convection in a cylindrical interlayer:

$$\mathrm{Nu} = \frac{2}{\ln \dfrac{1 + k\dfrac{h}{d}\cdot\dfrac{2}{f(\mathrm{Ra})}}{1-(1-k)\dfrac{h/d}{1+2h/d}\cdot\dfrac{2}{f(\mathrm{Ra})}}}, \tag{3}$$

where k is an empirical factor which signifies the effect of interlayer location and geometry (k = 0.5 corresponds to equal referred thicknesses of the thermal boundary layers δ_1 and δ_2), the Nusselt number Nu is defined with respect to the diameter of the inner cylinder ($\mathrm{Nu} = \alpha_{tot} d/\lambda$), and the Rayleigh number Ra is defined with respect to the interlayer thickness h. By an analogous analysis, we obtain the law of heat transmission by natural convection in a spherical interlayer:

$$\mathrm{Nu} = \frac{2 + 1/\dfrac{h}{d}}{\dfrac{2 + 1/\dfrac{h}{d}}{2 + \dfrac{f(\mathrm{Ra})}{k\dfrac{h}{d}}} + \dfrac{1-k}{f(\mathrm{Ra})\left(1 + 2\dfrac{h}{d}\right) - 2\dfrac{h}{d}(1-k)}}. \tag{4}$$

These formulas are conveniently analyzed in relative terms: by comparison with purely conductive heat transmission:

$$\varepsilon = \frac{\mathrm{Nu}}{\mathrm{Nu}_T},$$

for a cylindrical interlayer

$$\varepsilon = \frac{\ln\left(1 + 2\dfrac{h}{d}\right)}{\ln \dfrac{1 + k\dfrac{h}{d}\dfrac{2}{f(\mathrm{Ra})}}{1-(1-k)\dfrac{h/d}{1+2h/d}\dfrac{2}{f(\mathrm{Ra})}}}, \tag{5}$$

and for a spherical interlayer

$$\varepsilon = \left\{ \frac{2 + 1/\dfrac{h}{d}}{2 + \dfrac{f(\mathrm{Ra})}{k\dfrac{h}{d}}} + \frac{1-k}{f(\mathrm{Ra})\left(1 + 2\dfrac{h}{d}\right) - 2\dfrac{h}{d}(1-k)} \right\}^{-1}. \tag{6}$$

We will now examine the trends of relations (3)-(6) as their parameters approach extreme value. The conclusions will apply equally to cylindrical and to spherical interlayers.

As $\mathrm{Ra} \to \mathrm{Ra}_{cr}$ ($f(\mathrm{Ra}) \to 1$), the value of ε approaches unity, i.e., natural convection has no effect on the heat transmission.

As the interlayer curvature (i.e., the value of h/d) decreases, ε increases and approaches its corresponding value for a plane interlayer so that relations (5), (6) in the limit become $\varepsilon = f(\mathrm{Ra})$. This means that, at a fixed value of Ra, the relation ε(h/d) is a monotonic function: as h/d increases, ε decreases from its corresponding value for a plane interlayer (when h/d = 0) and approaches its asymptotic value

$\varepsilon = 1$ when $h/d \to \infty$. This conclusion can be easily verified experimentally with a constant gap h and varying cylinder (sphere) diameters.

If the diameter of the outer cylinder (sphere) is fixed and the diameter of the inner one is varied during the tests, then each $\varepsilon(h/d)$ curve goes through a maximum as a result of the opposing effects which the Rayleigh number and the curvature h/d have on the heat transmission, approaching the value $\varepsilon = 1$ asymptotically from the right and from the left. The left-hand branches of these curves reflect the decreasing significance of natural convection in the heat transmission, by virtue of the decreasing Rayleigh number (decreasing interlayer thickness), while the right-hand branches reflect the decreasing significance of natural convection with increasing curvature h/d. Changes in the law of heat transmission due to entering the Knudsen region, as the diameter of the inner shell is decreased, havenot been taken into account here. Maximum $\varepsilon(h/d)$ corresponds to $h/d \approx 1$.

An important conclusion follows from an analysis of the relation between heat transmission and interlayer curvature h/d at a fixed diameter of the inner shell and with the diameter of the outer shell varied, i.e., during transition from an enclosed interlayer to an infinite space. Function Nu(h/d) is shown in Fig. 1, based on calculations according to Eq. (3) for a cylindrical interlayer. The trend of the Nu(h/d) function for spherical interlayers is qualitatively analogous. The calculations were made with values of f(Ra) for a horizontal interlayer according to the data in [1-3], the temperature drop for the Rayleigh number equal to 10°C, with the thermophysical properties equivalent to those of ethyl alcohol, and with the coefficient k assumed equal to 0.5. The presence of an empirical factor k in formulas (3)-(6), the physical significance of which has been discussed earlier, makes these formulas somewhat indeterminate. This difficulty is not so great, however, inasmuch as the numerical value of k should in all cases differ little from 0.5. For comparison, in Fig. 1 has been plotted the curve for pure conduction. As h/d increases, according to the graph, the departure from the conduction curve at higher values of h/d begins earlier as d increases. The other extremum, a much flatter one and determined by the stabilizing effect of the interlayer curvature, precedes a leveling off to a constant value of Nu which will be demoted by Nu* and which corresponds to heat transmission from a body inside an infinite volume. According to (3) and (4), saturation occurs when the dimensions of the outer shell are such that f(Ra) becomes proportional to $Ra^{1/3}$ and the following inequalities hold true:

for a cylinder

$$(1-k)\frac{h/d}{1+2h/d}\,\frac{2}{f(\mathrm{Ra})} \ll 1, \tag{7}$$

for a sphere

$$\frac{1-k}{f(\mathrm{Ra})(1+2h/d)-2h/d(1-k)} \ll \frac{2+\dfrac{1}{h/d}}{2+\dfrac{f(\mathrm{Ra})}{kh/d}}\,. \tag{8}$$

The formula for Nu* will be found with the aid of data on heat transmission through plane interlayers: f(Ra) for $Ra > 10^6$ in horizontal and vertical plane interlayers is adequately well expressed $0.07(Ra)^{1/3}$ [1, 3]. With k = 0.5, relations (3) and (4) yield:

for a cylinder

$$\mathrm{Nu}^* = \frac{2}{\ln\left(1+\dfrac{1}{0.07\,(\mathrm{Ra}^*)^{1/3}}\right)}, \tag{9}$$

for a sphere

$$\mathrm{Nu}^* = 2 + 0.14\,(\mathrm{Ra}^*)^{1/3}, \tag{10}$$

where $Ra^* = (g\beta\Delta T/\nu a)d^3$ is now defined with respect to the diameter of the inner cylinder (sphere). The diameter of the outer shell, which may be regarded as infinitely large here, is determined from inequality (7) or (8) with $f(Ra) = 0.07(Ra)^{1/3}$.

We will now examine the feasibility of using these relations for a generalization of test data on heat transmission by natural convection in interlayers and in infinite volumes.

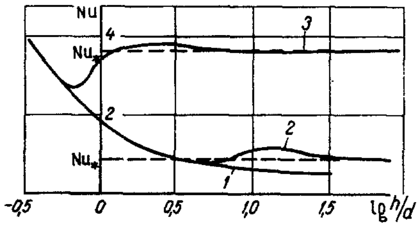

Fig. 1

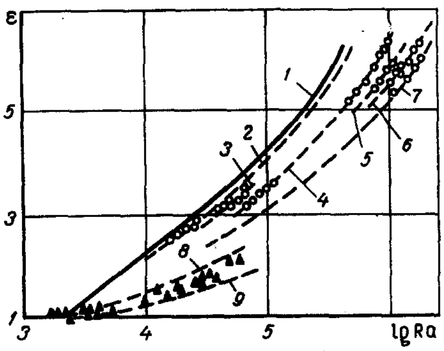

Fig. 2

Fig. 1. Relation Nu(h/d) at a fixed diameter of the inner cylinder: 1) pure heat conduction; 2) d = 0.2 mm; 3) d = 2 mm.

Fig. 2. Relation ε(Ra) for horizontal cylindrical interlayers; the points represent test values, the dashed lines calculations for various values of h /d: the darkened points represent data from [5] with h/d varied from 15 to 60, and the open circles data from [4] with h/d = 0.5-4.5; 1) relation ε(Ra) for a horizontal plane interlayer from [1, 2, 3]: 2) h/d = 0.5; 3) 0.93; 4) 1.5; 5) 2.36; 6) 3.5; 7) 4.5; 8) 12.5; 9) 50.

Data on heat transmission in horizontal cylindrical interlayers, taken from the technical literature [4, 5], are shown in Fig. 2. The curves represent calculations according to formula (5) with the appropriate values of the parameters, with the values of f(Ra) for a plane interlyaer [1-3], and with k = 0.4. The test data, which pertain to a very wide range of h/d and Ra, are adequately well described by Eq. (5). It is evident here that the test results for a small interlayer curvature (low values of h/d) are close to the test results on heat transmission in a plane interlayer, while the test values for a large curvature are much lower than the corresponding values for a plane interlayer.

There is a vast amount of data available on heat transmission by natural convection from a cylinder inside an infinite volume, with Ra* varying from 10^{-4} to 10^{12}. Empirical formulas have been proposed for approximating these test results by power functions in Ra* with different proportionality factors and power exponents over four ranges of Ra* [8, 9]: thus, the power exponent would be 0, 1/8, 1/4, and 1/3, respectively. Relation (9) yields an adequate description of the test data over the entire range $10^{-4} < Ra^* < 10^{12}$. This is illustrated in Fig. 3, where the solid lines represent calculations according to (9), while white and black dots represent test data from [8, 9] pertaining to horizontal and vertical cylinders, respectively. Because of the logarithm in (9), Nu* is only weakly dependent on Ra* within the range of low Ra* values. When Ra* varies from 10^{-1} to 10^{-4} (through three order of magnitude), for instance, then Nu* changes from 0.57 to 0.35 (only by 38%). Such a weak dependence at low Ra* values may pass unnoticed during tests because of the limited measurement precision. This could explain why Nu* was assumed constant and equal to 0.45 in [8, 9] for $Ra^* < 10^{-3}$.

The test data from [8, 9] on heat transmission from a sphere inside an infinite volume are shown in Fig. 4. According to the graph, they fit very accurately on the curve which represents Eq. (10). For the range of low Ra* values there are no data available but, on the basis of studies concerning the evaporation rate of liquid droplets [10, 11], it can be said that Nu* ~ 2 when Ra* ≈ 1. For comparison, on the same diagram has been plotted the Nu*(Ra*) curve for a cylinder according to (9). Apparently, the curves begin to diverge appreciably only when $Ra^* < 10^5$, i.e., within the range for which no test data on heat transmission from a sphere are available. This has also led to the conclusion in [8, 9] concerning the communality of the laws of heat transmission for a sphere and a cylinder in an infinite volume.

An analysis of heat transmission from a cylinder to a transverse stream with a low Reynolds number should reveal a stabilizing effect of natural convection, i.e., Nu(Re) should tend to a finite value and not to zero when Re → 0. Accordingly, the study in [13] of heat transmission by forced convection at a wire 0.1 mm in diameter in a nitrogen stream within the 100-1000°C temperature range has yielded the relation $Nu = 0.45 + 0.55Re^{0.5}$. Formula (9) for Nu* yields for the same conditions 0.42-0.48, values close to the first term of that relation.

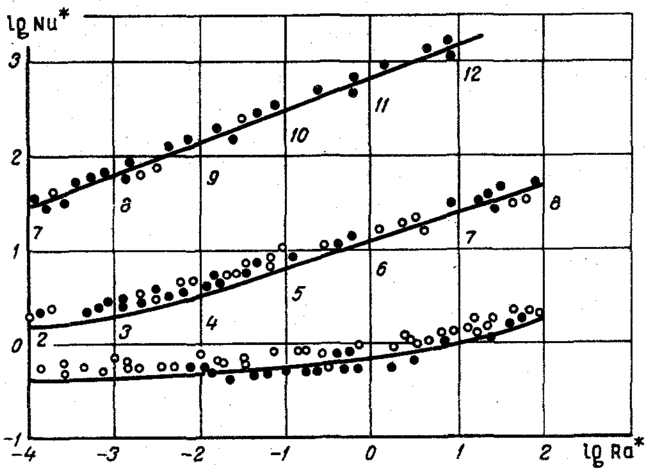

Fig. 3. Relation Nu*(Ra*) for a cylinder: the dots represent test values according to [8, 9]; the solid lines represent calculations according to formula (9).

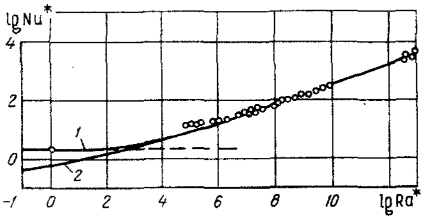

Fig. 4. Relation Nu*(Ra*) for a sphere according to data in [8, 9] (dots represent test points), calculated according to formula (10) (1); analogously for a cylinder according to formula (9) (2).

An analysis of these results leads to some conclusions concerning the well-known "hot-wire" method of thermal conductivity measurements in gases and liquids [12]. Researchers who use this method try to eliminate the effect of natural convection on the overall heat transmission by reducing the gap width h, i.e., the curvature h/d. If all these conclusions are correct, then one could obviously recommend the other extreme condition: large values of h/d. In that case, taking advantage of the very weak relation between Nu* and Ra* within the range of small Ra* values, one can measure the thermal conductivity according to (9) with sufficient accuracy – even if the thermophysical properties of the medium (viscosity and thermal diffusivity) are known only roughly.

It should be noted, in conclusion, that this analysis of heat transmission by natural convection in cylindrical and spherical interlayers has been based on heat transmission data for plane interlayers and, essentially, with the effect of curvature only taken into account. For this reason, the major aspects of the results must be verified by special tests and the relations must be refined. The effect of curvature on the heat transmission was examined earlier, but experimentally only (e.g., in [4, 14]). This, of course, renders these laws applicable only to the given range of parameter variation and does not make it feasible to analyze changes in heat transmission during extrapolation to extreme conditions.

NOTATION

g	is the acceleration of gravity;
β	is the thermal volume expansivity;
ΔT	is the temperature difference between interlayer boundaries;
ν	is the kinematic viscosity;
a	is the thermal diffusivity;
h	is the interlayer thickness;
w	is the velocity of fluid;
λ	is the thermal conductivity.

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